2021南开大学泛函分析系列报告

本次系列报告得到中国科协“海峡两岸青年数学家常态化论坛”项目的支持

日期	时间	报告人	题目
8月28日上午	腾讯会议/voov meeting: 644 698 954		
	9:00-9:40	黄毅青	Linear angle preservers of Hilbert C^*-modules
	9:40-10:20	方向	Random Dirichlet Multipliers
	10:20-11:00	蔡明诚	Mappings preserving trace of products of matrices
	11：00-11:40	王雅书	How $P_1(G)$ determines a finite group G

报告摘要

(1) **姓名:** 方向

单位: 中央大学

报告题目: Random Dirichlet Multipliers

摘要: This talk concerns the regularity of random multipliers on the Dirichlet space over the unit disk. In 1993, Cochran-Shapiro-Ullrich proved the following elegant result: For any $f(z) = \sum_{n=0}^{\infty} a_n z^n \in \mathcal{D}$, the Dirichlet space over the unit disk, almost all of its randomizations

$$(\mathcal{R}f)(z) = \sum_{n=0}^{\infty} \pm a_n z^n$$

are multipliers of $\mathcal{D}$. The purpose of this talk is to extend it in three directions, all inspired by the 1930 theorem of Littlewood on random Hardy functions. This is part of our ongoing project on Banach spaces of random analytic functions. (Joint work G. Cheng, C. Liu and Y. Lu at Dalian University of Technology)

(2) **姓名:** 蔡明诚

单位: 台北科技大学

报告题目: Mappings preserving trace of products of matrices

摘要: We show that two maps ϕ and ψ on the set of positive definite matrices satisfying

$$\operatorname{tr}(\phi(A)\psi(B)) = \operatorname{tr}(AB)$$

if and only if there exists a nonsingular matrix M such that $\phi(A) = M^*AM$, $\psi(A) = M^{-1}AM^{-*}$; or $\phi(A) = M^*A^tM$, $\psi(A) = M^{-1}A^tM^{-*}$. In addition, we characterize maps $\phi_1, \ldots, \phi_m$ $(m \geq 3)$ on the set of positive definite matrices satisfying

$$\operatorname{tr}(\phi_1(A_1)\cdots\phi_m(A_m)) = \operatorname{tr}(A_1\cdots A_m).$$

Moreover, the maps $\phi_1, \dots, \phi_m$ on the set $\mathcal{S}$ preserving the similar trace equality in $\mathcal{S}$ are also characterized, where $\mathcal{S}$ denotes the set of complex, Hermitian, symmetric, positive, doubly stochastic, and diagonal matrices, respectively.

(3) **姓名:** 王雅书

单位: 中兴大学

报告题目: How $P_1(G)$ determines a finite group G

摘要: Let G be a finite group and let $P_1(G)$ denote the set of all norm one positive definite functions on G. That is,

$$P_1(G) = \{\langle \pi(\cdot)\xi, \xi\rangle | \pi : G \to \mathcal{U}(H) \text{ unitary representation, } \xi \in H, \|\xi\| = 1\}.$$

In this talk, I will present that $P_1(G)$ determines G in many situations. We can tell if G is abelian, cyclic, simple, perfect, solvable, supersolvable or nilpotent via $P_1(G)$. Especially, then G is abelian, we can determine $G \cong \prod_j Z_{P_j^{r_j}}$ as a direct product of its cyclic subgroups of prime power orders.

(13) **姓名:** 黄毅青

单位: 中山大学

报告题目: Linear angle preservers of Hilbert C^*-modules

摘要: Let x, y be two vectors in a (real or complex) Hilbert C^*-module $\mathcal{H}$ over a C^*-algebra $\mathcal{A}$. The angle $\angle(x, y)$ between x and y can be defined in several way. When $\mathcal{A} = C_0(X)$ is a commutative C^*-algebra, in other words, $\mathcal{H}$ is a continuous field of Hilbert spaces over a locally compact space X, we define the cosine of the angle, $u = \cos\angle(x, y) \in C(X)$, by the equation

$$|\langle x, y\rangle| = |x||y|u.$$

We show that if $T : \mathcal{H} \to \mathcal{K}$ is a linear module map between two Hilbert $C_0(X)$-modules preserving (cosines of) non-flat angles, then $T = \alpha J$

for a bounded, strictly positive and continuous scalar function α on X and a module into isometry $J : \mathcal{H} \to \mathcal{K}$.

For a Hilbert C^*-module $\mathcal{H}$ over a non-commutative C^*-algebra A, we study the cases when $u = 0$ or $u = 1$, namely, the orthogonality or the parallelism of two vectors x, y in $\mathcal{H}$. While the linear orthogonality p-reservers are well studied in previous works, the problem of describing linear parallelism preservers seems to be rather difficult. After presenting some intrinsic properties of different versions of parallelism, we show that, considering the example of $\mathcal{H} = A = M_n$, any bijective linear map $T : M_n \to M_n$ of a matrix algebra preserving various types of parallelism assume the form $TA = \alpha UAV$ for some positive scala α and unitary matrices U, V in M_n.

This is a joint work with Luoyi Shi (石洛宜) of Tiangong University